Bed Balancing in Surgical Wards via Block Scheduling

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ARTICLE INFO	ABSTRACT

Background: Operating room (OR) planning involves the creation of a "master surgical schedule" in which surgeons are assigned to specific operating rooms (ORs) on specific days of a week. The master schedule is typically one or two weeks long repeatable for several months.

Objectives: The purpose of this study was to recommend using a mathematical program to generate a rotation in a way that the limited operating room capacity could be distributed based on smoothing expected demand for in-patient beds.

Patients and Methods: This study concentrated on the service-level scheduling at Sunnybrook Health Sciences Centre in Toronto, Canada, to build such a model. We assumed that the number of blocks (days) for each surgeon was given, and that the expected case-mix for each surgeon was chosen by random sampling based on historical data. The goal was to assign surgeons to the blocks so tat bed occupancy in the wards would become as stable as possible during the week. The planning problem was first formulated as a stochastic integer programming. Then, an approach with combination of Monte Carlo simulation and Premium Solver provided an approximate solution.

Results: The integer program provided scheduled OR number and day of the week for each surgeon, corresponding to the sample. The final result of model, approximated by the proposed method, was the maximum number of beds for each surgical service throughout the week. These were the required bed capacities to handle demands for surgeries.

Conclusions: An Integer Programming was presented to schedule OR and day of surgery for each surgeon with restrictions on the available ORs and required number of blocks. The problem was quickly solved using Premium Solver. The reliability of the results was highly dependent on the data. Another fundamental restriction for implementation of the results was to convince surgeons to accept changes in the schedules. The surgeon preferences might be included in the model constraints for more acceptable results.

▸ *Implication for health policy/practice/research/medical education:*
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1. Background

In recent years, there has been growing number of research studies aimed at planning of operating room (OR) activities. This is due both to the high cost of surgical facilities and the impact of their activities on the demand of hospital services and waiting time (1, 2). Planning of surgical activities (pre-, peri- and post-operative) is not an easy task because of large number of decision variables and the uncertainty. Uncertainties in the OR environment arise from emergency patient arrivals, variable surgery duration times and patient length of stay (LOS), or consumption of other resources (1-3). If these uncertainties are not taken into account, the hospital could face serious service quality problems generating unexpected costs .There are two well-known systems for surgery scheduling: block scheduling and open scheduling. In block scheduling system, the time of individual surgeon or surgical groups are assigned periodically (typically weekly or monthly) in a particular OR. Surgeons book the cases into their assigned time subject to fitting to mean duration of cases within the scheduled time. In open scheduling system, the intention is to accommodate all patients. The surgeons submit their cases until the day of surgery to schedule all cases in ORs. Individual surgeries are allocated in ORs to create a pre-operative schedule. In many hospitals, surgical scheduling is carried out as follows. At the beginning of each planning cycle (e.g. semi-annually), blocks of OR capacity (typically in full days) are allocated to various surgical services. Part of OR time may be reserved for emergency surgeries. A master surgical schedule in block scheduling methodology assigns a fixed amount of OR hours in a given day and time (block) to a particular surgeon or surgical service (3). Scheduling of ORs consists of several phases. In the first phase, the number of blocks is assigned to each service. This may be performed according to surgical waiting lists, marginal revenue, or priority score for a service. In the second phase, days and ORs are assigned to specific surgeons and scheduled for services. Finally, at the operational weekly level, patients are selected and sequenced within each block.

1.1. The Sunnybrook Health Sciences Centre

Sunnybrook Health Sciences Centre is a large healthcare provider located within the Toronto area that provides full range of acute care services ranging from neurology to orthopedic care. In the Sunnybrook operating department (OD) there are twenty ORs in which up to eighty surgeons are scheduled. Surgeons handle a mix of emergency, elective, and urgent cases as both in-patient as well as day surgery. This study focused on scheduling of elective and urgent in-patients while considering post-operative bed requirements as a resource constraint (4). Patients are admitted through the Same Day process for elective

surgeries. From the Same Day Department, patients are transferred to operating rooms that are distributed in M-Ground, M-2, and Burn-Unit areas. *Figure 1* shows the patient flow through the OD. Patients recover in the first and sometimes the second stage post-anesthetic care unit (PACU) beds that are located in either M-2 or Same Day unit. After the recovery, patients are moved to one of four post-operative unit or short-stay unit beds, or they are discharged. As patients progress through this process, their care may require the use of up to five distinct bed types with limited capacities: pre-operative anesthesia block rooms, ORs, first stage PACU beds, second stage PACU beds, and finally post-operative ward beds. Ward beds are the primary constraint on the procedures. As a result, an increasing number of procedures are being cancelled due to the absence of sufficient ward beds for post-operative recovery of patients. This leaves OR times free and makes surgeons idle and unable to provide patient care (4). Manager of the OD is responsible to assign operating rooms, nurses, and other resources to different services offered in the hospital. This service-level schedule is being updated monthly, and specifies, on daily basis, available operating rooms for each service. Surgeon-level schedule, the second level of scheduling, is created individually by each service when the services receive their surgical blocks. At this point, the services will allocate specific surgeons, procedures, and patients in available times. Each surgical division uses the master schedule to book its surgeons and operations into the allotted surgical blocks. There are several scheduling issues need to be accounted at surgeon-level schedule. Surgeons collaborate for OR time to complete their procedures. Noteworthy is the clear distinction between service- and surgeon-level schedules. At service-level, the manager distributes OR time between surgical divisions; this schedule is so constrained in terms of service assignment options. At surgeon-level, surgeons claim their time in OR's, which is based on total allotted resources they have been assigned on macro schedule: in this schedule, the OD manager has no control over which surgeons work at which times (on micro schedule) but is responsible for providing available OR time to surgeons (4).

1.2. Purpose of the Study

Cancelled procedures are equally detrimental to patients, surgeons, and hospital administration. Patients have to wait longer to receive care, and the hospital administration does not receive full utility from invested resources in operating rooms. It is particularly disruptive for surgeons who are paid on a service-based fee. There is clear evidence that absolute bed capacity is not the leading cancelling issue of procedures. Reports on bed census show that bed utilization in ORs ranges anywhere from 67% (periods of complete use) to 112% (periods of significant underutilization of available ward beds)(5). Utili-

zation above 100% indicates admitted patients occupying stretchers in the Emergency department (ED). Bed utilization varies significantly throughout the week; although this may seem unavoidable due to inherent randomness in the system, there are actually several factors, including the number of each type of surgery and the patient's LOS, which can be controlled during the scheduling phase. From these factors, admission to and discharge from ward beds can be estimated. Litvak and Long pointed out that while non-elective cases contributed to variability in hospital environments, an important part of variance could be controlled by applying scheduling policies to elective cases (6). Although the number of beds occupied by a surgeon's patients is largely determined by doctor's case-mix and the LOS, the total number of cases along with their resulting LOS, for a given day, can be controlled by scheduled services and surgeons. Calichman emphasized that the key to improving OR schedule performance is to use the historical relationship between surgical categories and their LOS distributions in post-operative care (7). Chow and Taylor applied a local search algorithm in a simulation model of OD at Sunnybrook Health Sciences Centre, for the service- and surgeon-level schedules to lower variability in bed utilization (4). They showed significant differences between services, and between surgeons within almost all services. This identifies clear distinctions that make rescheduling based on the influences of two factors potentially worthwhile: the number of cases per block and the length of stay per case among the surgeons. This study concentrated on the service-level scheduling for surgeons at Sunnybrook Health Sciences Centre for which the number of blocks for surgical services was given and the case-mix groups for surgeons were known or could be predicted. The main question of this research was that how a mathematical program could be developed to control and maximally flatten the bed occupancy in the wards during the week. The second question was whether it was possible to decrease the cancellation rate for surgeons by changing the day of in-patient surgeries. The final question was that how one could estimate the minimum required bed capacity to handle demands of surgeries.

1.3. Assumptions and Limitations

We assumed that the patient-mix for each surgeon could be modeled using historical data. By analyzing past year surgical records, we could determine distribution of the number of patients in each block and the case-mix patterns. The model employed these distributions to construct stochastic sample surgical days for each surgical block. We did the same for patient LOS for each case-mix group (CMG); we randomly computed the length of stay based on patient's history. Without loss of generality, we assumed that every surgeon acquired at least one block (one full day) per week; for more than one block,

we created multiple copies of the surgeon. A limitation to our model was our assumption that the surgeon had the same case-mix in each surgical block; it was quite possible that a surgeon would perform different types of surgery in different blocks. This would require a modification to the model to explain how one would evaluate historical patterns. When a surgeon acquired a half-day, it was assumed that he/she was already pre-matched with another half-day surgeon. From the modeling perspective, these surgeons and their respective case-mixes were considered as one surgeon and their historical data were merged. Another limitation was a fundamental restriction that prevented administrative staff at Sunnybrook Health Sciences Centre from scheduling surgeons to work at specific times. Convincing surgeons to accept changes for the sake of system-wide improvements would be a challenge: they had a series of commitments preventing them to work in certain days or times. Clinic and office working times as well as private occasions could affect on availability of surgeons. These restrictions could seriously limit the schedule and significantly increase the variability of bed utilization. The days on which surgeons prefer not to operate may be considered as constraints of this model. The model was primarily applicable for Monday through Friday. However, Sunnybrook Health Sciences Centre has recently begun the scheduling of some elective procedures (mainly orthopedic) on Saturday. The Saturday blocks were much more variable than other days since surgeons varied and the blocks could not move to other days. Nevertheless, patients contributed to ward LOS, and therefore we needed to include their patterns in our balancing efforts; by the way, Saturdays and Sundays were excluded from the scheduling. Finally, although OD database records the patient types, there was no field in database of surgical wards to distinguish elective from emergency cases. However, as mentioned above, emergency and urgent cases were included in the historical surgery data, so they were included in the model implicitly.

1.4. Literature Review

Previous researchers considered a variety of objectives and different aspects and assumptions to plan the surgical schedule. They mainly focused on the surgeon-level scheduling, and the third phase considering OR utilization. Testi *et al.* brought together some planning elements for elective surgical activity into an approach that was hierarchically broken down into three main phases in a way that the solution of one phase was used as input data for the next phase (8). In the first phase, the number of sessions to assign to each ward was selected. In the second phase, sessions were scheduled on the days of the planning period. Finally, in the third phase, the question of how one can select patients on in each session for the operation was resolved. Some research reviews on

the second phase and mostly on service-level planning are performed. Recently, few literature surveys reviewed most of researches in the area of surgical scheduling. Cardoen *et al.* focused the review on the manuscripts that explicitly incorporated planning and scheduling considerations and differentiated between strategic, tactical, and operational approaches (1). May *et al.* reviewed six categories: capacity planning; process reengineering/redesign; surgical services portfolio; procedure duration estimation; schedule construction; and schedule execution, monitoring and control, while focused on the various methodologies used (2). Guerriero and Guido surveyed papers with more interesting mathematical models and quantitative approaches to address OD management (3). They reviewed the articles more analytically, compared to other researches, by giving the key idea . Zhang *et al.* developed a mixed integer programming approach to minimize in-patient LOS waiting for allocating OR capacity to specialties in-patient for their surgeries (9). Blake *et al.* presented an integer programming model that minimized the weighted average undersupply of OR hours (10). The produced master surgical schedule (MSS), with a week horizon, was then extended to cover all the weeks of considered time horizon. Vissers *et al.* studied the problem of master surgery schedule while considering multiple resources; they formulated the problem as a mixed integer programming and solved it heuristically (11). Ozkarahan proposed a goal programming model for the assignment of surgical operations among multiple operating rooms with the aim of preventing over- or under-capacity loading (12). Efficient OR scheduling was further complicated by the variability that was inherent in surgical procedures, which decreased the utilization level. Van Oostrum *et al.* dealt with the variability of emergency surgery by assigning planned slack times to ORs (13). They considered stochastic surgery duration time, but deterministic LOS requirements for each type of patient. Lamiri *et al.* formulated a stochastic integer programming taking into account uncertainty related to surgery duration time and emergency surgery demand (14). They presented an "almost" exact method combining Monte Carlo simulation and mixed integer programming, and investigated its convergence properties.

2. Objectives

This study concentrated on the second phase of OR scheduling for surgeons as well as service-level at Sunnybrook Health Sciences Centre to achieve bed balancing. In the study, we investigated how variability in bed capacity utilization could be reduced through effective scheduling of services and surgeons.

3. Patients and Methods

OR planning involves the creation of the master surgi-

cal schedule, in which surgical services will be assigned to specific operating rooms on specific days of the week can be used repeatedly by the hospital until the capacity or demand changes. The cyclic schedules are patterns that repeat after a certain period of time. The goal of this study, as a part of master surgical schedule (MSS), was to generate a two-week rotation where all surgeons were assigned to blocks, in a way that the limited OR capacity was distributed based on smoothing expected demand for resources (primarily the beds). This research directly addressed elective surgery planning based on the assumption that patients' LOS were stochastic. The number of patients for each surgeon was generated randomly according to the historical data, which included both emergency and non-emergency in-patients. It was assumed that, based on requirements and priorities, a prescribed number of blocks have already been allocated to each service in advance. Therefore, the problem was the second phase of scheduling, which was defined as follows: given the number of blocks for each surgeon, scheduling the blocks and surgeons to specific operating rooms by which expected bed occupancy for each service would be balanced during the days of a week.

3.1. The Research Method

For each CMG in the historical data, we determined the probability distribution for patient LOS in the surgical wards. Goodness of fit tests were performed using the Stat-Fit package (15). The uncertainty of the LOS made the planning for wards' bed utilization complicated. The surgeon scheduling could not be considered as a deterministic optimization, but as a two-stage stochastic programming with recourse (16), in which some parts of the mathematical model were stochastic. At the first stage, random samples of independent replications of random variables were generated and consequently the stochastic variables were approximated by the average values (16). Then, the approximate model would be solved in the second stage. The core idea was to use random samples of parameters or inputs to explore the behavior of a complex process. Monte Carlo simulation is a versatile method for analyzing the behavior of some activity, plan, or process that involves uncertainty. Generating random samples and consequent approximation of the expectations by the corresponding average is the heart of Monte Carlo method (16). Therefore, different sets of random samples were generated to represent the uncertain variables such as the number of elective surgeries for each surgeon and LOS of each patient at surgical ward. Then, the corresponding expectations were approximated by their sample averages. The first stage of the method used Monte Carlo simulation. Consequently, we were able to write deterministic integer programming equivalent of the problem using the sample average approximation. It was shown that, by increasing the sample size, the opti-

mal solution of deterministic equivalent of the problem would converge to the optimal solution of the main stochastic integer program (14, 16).

3.2. Mathematical Model

The planning horizon consisted of 26 weeks, each week of 5 days, and each day of 1 session or block. We needed to estimate the required number of beds in any day of the week for each ward based on random samples of patients and their LOS generated for each surgeon according to experienced patterns. The objective function was the summation of required beds that would be minimized. Smoothing required number of beds could be retained by minimizing the maximum number of required beds on different days. Clearly, the model could be extended across all services with different priorities. The OR-surgeon scheduling problem was formulated as a stochastic integer programming based on corresponding data on CMGs, demands, and resources. The binary decision variables defined the day and OR allocated to the surgeons, which were subsequently used to define total number of required beds for planned patients at each service (who were operated by different surgeons in corresponding services) on a definite day of the week in order to form a simple equation. Some patients may stay longer than a week. Therefore, for example, a patient with a surgery on Monday who stays nine days uses, on average, two beds on Monday and Tuesday nights. This constraint was formulated knowing the maximum historical LOS for each service and using "mod 7" in the summation *(Appendix)*. Other constraints were formulated to limit one block for each OR per day, to assign for each surgeon his/her required number of blocks of OR time per week, and to ensure that no surgeon would get more than one block each day, and that surgeons

were assigned to appropriate ORs *(Appendix)*.

3.3. Data Requirements

The data used in this study, including the patients' pattern for surgeons and LOS, were extracted from available electronic record systems over a period of two years, 2007 and 2008. The patients' records were filtered for non-emergency in-patient surgeries, as these cases would represent controlled procedures that require a planned ward bed as a resource. The data were categorized by service as well as by surgeon. We only included current surgeons based on the 2008 Sunnybrook OR electronic record system. *Table 1* refers to the primary data of service name, number of surgeons, and specific ORs numbers. Then, since the surgeons' numbers of sessions per

Table 1. Number of Full-Time Surgeons and Available ORs for Services

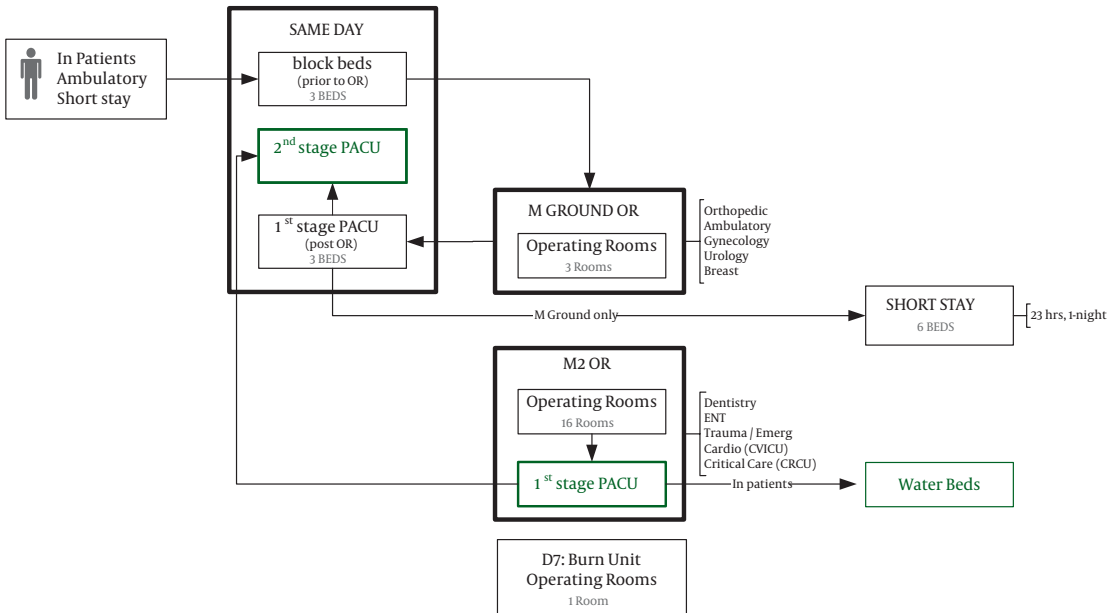|	Surgeons, No.	Specific OR Number
Cardiac Surgery	5	14, 15, 16, 18
Cardiology	2	2, 18
General surgery	8	1, 2, 4, 5, 8, 9
Gynecology	2	5, 9
Neurosurgery	4	11, 12
Ophthalmology	1	7
Oral Surgery	1	14a
Orthopedics	11	3, 4, 11, 20
Otolaryngology	2	11, 14a
Plastic Surgery	4	5, 9, 10, 11
Urology	4	2, 5, 6, 8, 9
Vascular Surgery	3	14
Burn	3	17

Figure 1. The Patient Flow Through OD, Sunnybrook Health Sciences Center The figure is extracted from the article by Chow E *et al.* with some changes (4).

week were randomly variables, they were approximated by their modes. Stat-Fit (15) was used to find the best distribution to model the number of cases per week. For all surgeons, a Poisson distribution was found to provide a good fit. Similarly, for LOS distributions, a Log Normal function with appropriate parameters was found to fit to almost all except two surgeons, who are indicated in *Table 2*. El-Darzi *et al.* (17) mentioned that the average length of stay in the acute care hospitals could be artificially high when some patients requiring the beds for long-term care or rehabilitation were kept waiting in the ward until availability of appropriate beds. We assumed

that the anomalies were probably due to extended LOS. Since a Log Normal distribution was considered as the best for fitting LOS (18), this distribution was used to predict LOS for all patients (with the corresponding parameters). *Table 2* contains a sample of random data generated for the surgeons. The Sunnybrook Health Sciences Centre also accepts emergency cases and usually reserves between two to four empty surgical rooms per day for these patients. An approach to deal with the uncertainty of emergency arrivals consisted of reserving planned blocks of ORs. Emergency cases represented 37% of total in-patient cases between April and August 2007 (4). In the

Table 2. One set of random data generated for the surgeons and the optimal schedule

Service	Surgeon	Blocks or Sessions, No.	Patients, No.	LOS Distribution [a], Mean ± SD	Session 1			Session 2		
					LOS for Patients [b]	OR	Day	LOS for Patients [b]	OR	Day
1	1	2	3	2.29 ± 0.82 [c]	14, 13, 11	16	2	12, 15, 14	14	1
1	2	2	3	2.34 ± 0.73 [c]	14, 12, 13	15	4	14, 14, 12	15	2
1	3	1	2	2.52 ± 0.62	15, 14	18	2	-	-	-
1	4	1	1	2.03 ± 1.06	13	18	1	-	-	-
1	5	1	2	2.24 ± 0.74	12, 11	15	1	-	-	-
2	6	1	1	2.04 ± 1.19	15	18	3	-	-	-
3	7	1	1	2.56 ± 1.1	24	1	3	-	-	-
3	8	1	1	2.69 ± 1.26	32	1	1	-	-	-
3	9	1	1	2.86 ± 1.14	33	1	5	-	-	-
3	10	1	1	2.63 ± 1.16	27	1	2	-	-	-
3	11	1	1	2.66 ± 1.34	34	1	4	-	-	-
5	12	1	1	1.74 ± 1.53	17	11	5	-	-	-
5	13	1	1	2.62 ± 1.56	47	12	5	-	-	-
5	14	1	1	2.18 ± 1.27	19	12	3	-	-	-
5	15	1	1	2.14 ± 1.22	17	11	2	-	-	-
5	16	1	1	2.52 ± 1.18	25	11	1	-	-	-
8	17	1	1	2.38 ± 1.06	18	20	4	-	-	-
8	18	1	2	2.43 ± 1.1	20, 22	3	4	-	-	-
8	19	2	2	2.39 ± 0.93	17, 16	4	1	16, 15	3	5
8	20	1	1	2.42 ± 0.73	14	20	5	-	-	-
8	21	2	2	2.36 ± 0.92	16, 15	3	2	15, 17	3	3
8	22	1	1	2.33 ± 0.92	16	11	3	-	-	-
8	23	1	1	2.58 ± 0.76	18	4	5	-	-	-
8	24	1	1	2.44 ± 0.99	19	4	4	-	-	-
8	25	1	2	2.32 ± 0.997	16, 17	3	1	-	-	-
8	26	1	1	2.43 ± 1.01	19	20	3	-	-	-
10	27	1	1	2.38 ± 1.13	20	11	4	-	-	-
10	28	1	1	2.74 ± 0.996	22	5	4	-	-	-
10	29	1	1	2.35 ± 1.29	23	5	2	-	-	-
10	30	1	1	2.94 ± 0.99	32	5	5	-	-	-
11	31	1	1	1.93 ± 1.43	19	6	1	-	-	-
12	32	1	1	2.15 ± 1.22	17	14	4	-	-	-
2	33	1	1	2.06 ± 1.11	14	2	1	-	-	-
12	34	1	1	2.05 ± 1.14	15	14	5	-	-	-

[a] Log Normal
[b] For each session
[c] The Log Normal function with appropriate parameters were found to fit almost all except two surgeons

available electronic record system, there were no fields to distinguish data of elective from emergent cases. However, the percentage of the ward beds occupied by emergency patients might be approximated by the Utilization Management Report, Sunnybrook Health Sciences Centre (19) to predict the number of available beds for elective in-patients. The MSS for different months of 2007-2008 could be employed to query resource availability, i.e. the feasible ORs for services. *Table 1* gives the number of ORs equipped with the surgical facilities appropriate for each service. For example OR number 7 was equipped for ophthalmology and OR number 17 for Burn. However, most types of surgeries were scheduled for most ORs, though it was the policy to schedule specific services in specific ORs because of their feeling that it would make the planning problems more precisely and efficiently solvable.

Table 3. Number of Required Beds for Services

	Occupied Beds in a Week by the Model, No.	Average (Approximate) Occupied Beds in 2008, No.
Cardiac surgery	23	26
Cardiology	8	42
General surgery	22	32
Gynecology	-	1
Neurosurgery	17	15
Ophthalmology	-	1
Oral surgery	-	0
Orthopedics	32	22
Otolaryngology	-	2
Plastic surgery	14	3
Urology	7	15
Vascular Surgery	5	8
Burn	-	14

4. Results

Integer programming model has been implemented using Microsoft Excel 2003, and Premium Solver produced by Frontline Systems (version 8). An Excel Visual Basic interface allows users to enter data, change model parameters, execute the model, and view the results such as scheduled OR and day for each surgeon. The interface also provides users in terms of adding, deleting, or modifying the constraints included in the model. Model constraints allow users to specify the type of OR that may be assigned to a service, surgeon availability, preferences, and other operational restrictions. For each surgeon, the number of patients has been randomly generated along with the appropriate LOS. *Table 2* presents the distribution parameters for patients, LOS for each surgeon, and one set of generated sample including number of blocks, number of patients, and the corresponding LOS for each surgeon. Considering available ORs for services *(Table 1)* and surgeons who

were scheduled *(Table 2)*, the model consisted of 715 binary variables and 218 constraints, which was solved quickly (in less than 2 minutes) by Premium Solver on a Dell personal computer running Windows XP. *Table 2* presents the scheduled OR number and specified day of the week for each surgeon, corresponding to the sample and provided by integer programming; the last column corresponds to the second block in the week for those surgeons who had two blocks per week. The final result of the model, which was approximated by the proposed method, would be the maximum number of beds for each surgical service during the week, as shown in *Table 3*, in which "-" stands for no planned number of beds was required. These were required bed capacities to handle demands for the surgery. The approximate actual number of occupied bed in different wards was averaged using weekly bed allocation sheets on 2008 that is shown in the last column of *Table 3* for comparison.

5. Discussions

In this research, as original contribution of the paper, a Stochastic Integer Programming model was developed to create a Master Surgical Schedule at Sunnybrook Health Sciences Centre by which bed occupancies in the wards were balanced during the week. The model considered uncertainty of demands for surgery and the LOS. The stochastic distributions fitted to the data were used to randomly sample required quantities. An Integer Programming was developed to schedule the OR and day of surgery for each surgeon with restrictions on the available ORs and required number of blocks for surgeons. The objective was to minimize the weighted sum of maximum bed requirements during the week. This would flatten the bed occupancy at the wards. On the other hand, the number of required beds to hospitalize predicted patients was estimated as constraints of the model, which would prevent cancellation of the surgeries (i.e. there was no shortage of beds anymore, if stochastic patterns would not change). In fact, based on the generated data (Appendix), the optimal number of beds (which was an intermediate variable in the proposed model) was appropriate estimation for the bed capacities. None of previous researches dealt with leveling the occupied beds at the wards. The reason of many surgery cancellations is the lack of empty beds after the surgery. The motivator of this research was the scheduling services' blocks to prevent the lack of resources at the wards. Belien *et al.* developed analytical models and heuristics for determining the number of blocks for each service, and building cyclic master schedules to minimize expected total bed shortage (20). Their research was the first attempt to reach the goal of bed occupancy leveling (3). The current paper also proposed an OR scheduling problem to regulate occupancy

pattern at the wards, but with different objective and formulation. Belien *et al.* defined a stochastic variable as "the number of patients staying in the hospital with definite days after one block of surgery by a surgeon", but used later its equivalents in their model: the sum-product of "the probability a patient stays with definite days after surgery by a surgeon" and "the number of patients operated by a surgeon" (20). The current research defined random variable as "the number of patients with a definite LOS for each surgeon" and directly used it in the proposed formulation, which made the model simpler. We were concerned about scheduling problem for surgeons but at service-level. Another difference was that in this paper, our objective was smoothing the bed occupancy in different surgical wards, separately. There were some conflicts between the number of occupied beds provided by the model and the actual number of occupied bed at the surgical wards. The roots of this discrepancy were:

• Some patients were not admitted for some wards such as cardiology through OD.

• The model counted only non-emergency patients, i.e. Burn.

• The number of surgeons did not match with what OD supervisor knew about the actual number.

It also seems that those reports we were consulting, did not necessarily tie back perfectly to OR utilization. Chow and Taylor found that expected resulting for bed utilization from simulation did not match with the number of beds (4). The reliability of the results highly depends on the data. Another fundamental restriction for implementation of results was to convince surgeons to accept changes in the schedules. The surgeon preferences may be included in the model constraints for more acceptable results. There are potential extensions that would make the model more realistic. First, a more in-depth analysis can be made into the mining of accurate data. Second, the objective function can be modified to smooth the use of other resources such as intensive care unit (ICU) beds. Third, it can be assumed that ORs that are more specific would be available for services that are more specific; so, less ORs will be scheduled for different services. Then, the main problem may be decomposed to several separated sub-problems, and therefore, can be solved more efficiently. The use of the proposed model is not limited to the specific situation in this case study. The model can be easily extended to other cases by including related resources and restrictions. Further research is required to develop an efficient heuristic solving procedure by exploring the model structure.

Appendix

The OR-surgeon scheduling problem is formulated as a stochastic integer programming based on corresponding data on CMGs, demands, and resources. The binary decision variables as:

$$(1) \quad x_{rdj} = \begin{cases} 1 & \textit{If surgeon j operates in OR r on day d}, \\ 0 & \textit{otherwise} \end{cases}$$

define the day and OR allocated for the surgeons, independent of random samples which have been generated for random variable (i.e. the number of patients with LOS of i days for surgeon j). Now, total number of required beds for planned patients at service s (who were operated by different surgeons in service s) on day t is:

$$(2) \quad b_s^k = \sum_{j \backslash operating-in-service-s} \sum_{d=1}^{5} \sum_{i=(t-d)\bmod 7+1}^{max\_los} p_j^k \sum_r x_{rdj}$$

in which, max_los stands for the maximum historical LOS for service s. Some patients may stay longer than a week. So, for example, a patient with surgery on Monday who stays nine days uses, on average, two beds on Monday and Tuesday nights. Hence, we use "mod 7" in the summation. The sample average approximation (and deterministic) integer programming to schedule day and OR for surgeons by which the required beds at surgical services are smoothed is:

$$(3) \quad \min \quad \sum_s w_s Z_s$$

$$(4) \quad s.t. \begin{cases} \frac{1}{K}\sum_{k=1}^{K} b_s^k \leq Z_s & \forall t=1,\dots,7, \forall s \quad (4) \\ \sum_j x_{rtj} \leq 1 & \forall t=1,\dots,5, \forall r \quad (5) \\ & \quad (6) \\ \sum_{l \in \delta_j} \sum_r \sum_t x_{rtl} = n_j & \forall j \in s \\ \sum_{l \in \delta_j} \sum_r x_{rtl} \leq 1 & \forall t=1,\dots,5, \forall j \in \Delta \quad (7) \\ x_{rtj} = 0 \quad o \quad 1 & \forall t=1,\dots,5, \forall s, \forall j \in s, \forall r \quad (8) \\ x_{rtj} = 0 & \forall r \notin R_s \quad (9) \end{cases}$$

in which Zs is an intermediate variable indicating the maximum number of required beds in service s throughout the week, n_j is the required number of blocks per week for surgeon j (normally 1), and w_s is the priority weight for services. The OD consists of different types of ORs according to the type of surgical procedures, and Rs is the set of ORs equipped for surgeries of service s. Δ is the index set of surgeons who require more than one session, and for each, δ_j is the indexes of virtual surgeons corresponding to surgeon j. Each virtual surgeon-copy is assumed to have the same case-mix in each block. The objective is to minimize the weighted sum of maximum bed requirements throughout the week. This will smooth the variability of occupied beds in services. The services and surgeons within the services are weighted based on the preferences of management.

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Authors' Contribution

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References

1. Cardoen B, Demeulemeester E, Belien J. Operating room planning and scheduling: A literature review. *Eur J Oper Res*. 2010;**201**(3):921-32.
2. May JH, Spangler WE, Strum DP, Vargas LG. The Surgical Scheduling Problem: Current Research and Future Opportunities. *Prod Oper Manage*. 2011;**20**(3):392–405.
3. Guerriero F, Guido R. Operational research in the management of the operating theatre: a survey. *Health Care Manag Sci*. 2011;**14**(1):89-114.
4. Chow E, Taylor A. Bed Utilization and Operating Room Scheduling at Sunnybrook Health Sciences Centre. Toronto: University of Toronto; 2008.
5. Bed Occupancy. Sunnybrook Health Sciences Centre; 2008; Available from: http://sunnybrook.ca/.
6. Litvak E, Long MC. Cost and quality under managed care: irreconcilable differences? *Am J Manag Care*. 2000;**6**(3):305-12.
7. Calichman MV. Creating an optimal operating room schedule. *AORN J*. 2005;**81**(3):580-8.
8. Testi A, Tanfani E, Torre G. A three-phase approach for operating theatre schedules. *Health Care Manag Sci*. 2007;**10**(2):163-72.
9. Zhang B, Murali P, Dessouky M, Belson. A Mixed Integer Programming Approach for Allocating Operating Room Capacity. *J Oper Res Soc*. 2009;**60**(5):663-73.
10. Blake JT, Dexter F, Donald J. Operating room managers' use of integer programming for assigning block time to surgical groups: a case study. *Anesth Analg*. 2002;**94**(1):143-8, table of contents.
11. Vissers JMH, Adan IJBF, Bekkers JA. Patient mix optimization in tactical cardiothoracic surgery planning: a case study. *IMA J Manage Math*. 2005;**16**(3):281–304.
12. Ozkarahan I. Allocation of surgeries to operating rooms by goal programing. *J Med Syst*. 2000;**24**(6):339-78.
13. van Oostrum JM, Van Houdenhoven M, Hurink J, Hans E, Wullink G, Kazemier G. A master surgical scheduling approach for cyclic scheduling in operating room departments. *OR spectrum*. 2008;**30**(2):355-74.
14. Lamiri M, Grimaud F, Xie X. Optimization methods for a stochastic surgery planning problem. *Int J Prod Econ*. 2009;**120**(2):400-10.
15. Geer Mountain Software Corporation. stat::fit. Version 2 ed. South Kent, CT; 2003.
16. Shapiro A, Homem-de-Mello T. A simulation-based approach to two-stage stochastic programming with recourse. *Math Program*. 1998;**81**:301-25.
17. el-Darzi E, Vasilakis C, Chaussalet T, Millard PH. A simulation modelling approach to evaluating length of stay, occupancy, emptiness and bed blocking in a hospital geriatric department. *Health Care Manag Sci*. 1998;**1**(2):143-9.
18. Marazzi A, Paccaud F, Ruffieux C, Beguin C. Fitting the distributions of length of stay by parametric models. *Med Care*. 1998;**36**(6):915-27.
19. Master Surgical Schedule. Sunnybrook Health Sciences Centre; 2007-8; Available from: http://sunnybrook.ca/.
20. Belien J, Demeulemeester E. Building cyclic master surgery schedules with leveled resulting bed occupancy. *Eur J Oper Res*. 2007;**176**(2):1185–204.